MATHEMATICS

TOPIC: TRIGONOMETRY
GRADE 10

CAPS ALIGNED

TRIGONOMETRY - DEFINITIONS
(Extended for $0° \leq \theta \leq 360°$)

www.nbmmaths.co.za

TRIGONOMETRY– Grade 10

1. Trigonometry

1. Define the trigonometric ratios sin θ, $\cos\theta$ and $\tan\theta$ using the right – angled triangle
2. Extend the definitions of sin θ, $\cos\theta$ and $\tan\theta$ for $0° \leq \theta \leq 360°$
3. Define the reciprocal of the trigonometric ratios $\text{cosec}\,\theta$, $\sec\theta$ and $\cot\theta$, using the right-angled triangles (these three reciprocals should be examined in Grade 10 only)
4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0°, 30°, 45°, 60°, 90°\}$
5. Solve two-dimensional problems involving right-angled triangles (See Term 3)
6. Solve simple trigonometric equations for angles between 0° and 90°
7. Use a diagram to determine the numerical values of ratios for angles from $0°$ to $360°$

2. Trigonometry (2D)

1. Solve two-dimensional problems involving right- angled triangles
2. Problems in two dimensions

3. Examination Guideline

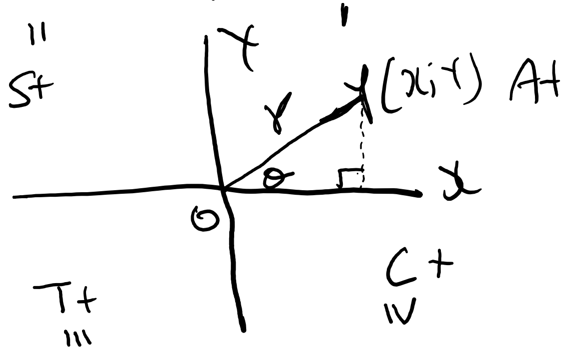
1. The reciprocal ratios cosec θ, sec θ and cot θ will be explicitly tested in all aspects: definitions, function values and equations.
2. While the focus of trigonometric graphs is on the relationships, the characteristics of the graphs will also be examined.

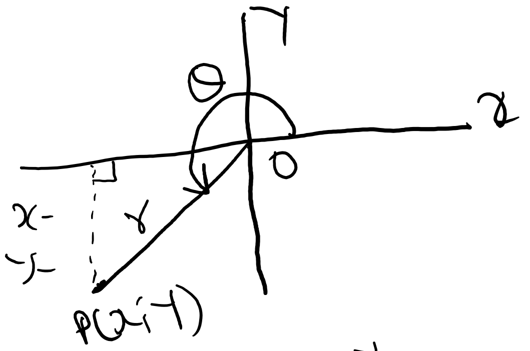
Trigonometry (Definitions Extended for 0°≤θ≤360°) - GRADE 10
Exercises - A

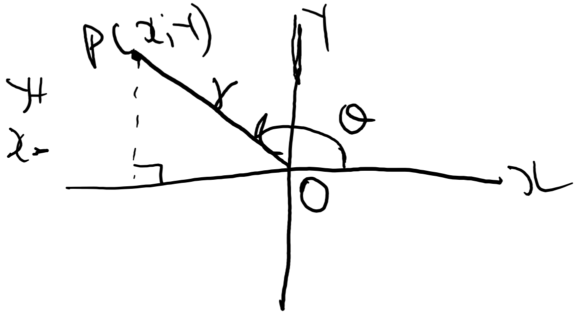
Notes: Toolbox

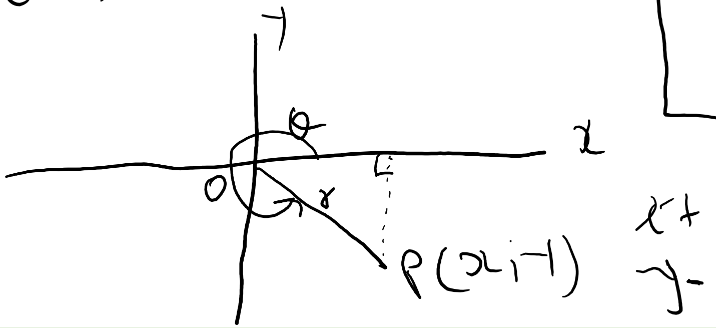
Definitions → in terms of x, y & r.

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ & $\tan\theta = \frac{y}{x}$









Theorem of Pythagoras

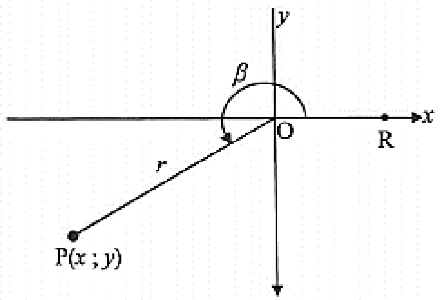
$x^2 + y^2 = r^2$

Trigonometry (Definitions Extended for $0° \leq \theta \leq 360°$) - GRADE 10
Exercises - A

Exercise A

QUESTION 5

5.1 In the diagram below, $P(x\ ;\ y)$ is a point in the third quadrant. $R\hat{O}P = \beta$ and $17\cos\beta + 15 = 0$.



5.1.1 Write down the values of x, y and r.

5.1.2 WITHOUT using a calculator, determine the value of:

(a) $\sin\beta$

(b) $\cos^2 30°.\tan\beta$

5.1.3 Calculate the size of $R\hat{O}P$ correct to TWO decimal places.

Solution

5.1.1) $17\cos\beta + 15 = 0$

$\cos\beta = -\frac{15}{17}$ $\left(\frac{x}{r}\right)$

$x^2 + y^2 = r^2$

$(-15)^2 + y^2 = (17)^2$

$\sqrt{y^2} = \sqrt{64}$

$y = \pm 8$

$\therefore y = -8$

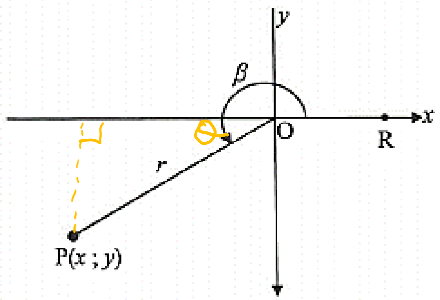
$\therefore x = -15,\ y = -8$ & $r = 17$

Trigonometry (Definitions Extended for $0° \leq \theta \leq 360°$) - GRADE 10
Exercises - A

Exercise A

QUESTION 5

5.1 In the diagram below, $P(x\ ;\ y)$ is a point in the third quadrant. $R\hat{O}P = \beta$ and $17\cos\beta + 15 = 0$.



5.1.1 Write down the values of x, y and r.

5.1.2 WITHOUT using a calculator, determine the value of:

(a) $\sin\beta$

(b) $\cos^2 30°.\tan\beta$

5.1.3 Calculate the size of $R\hat{O}P$ correct to TWO decimal places.

5.1.2 (a) $\sin\beta = \left(-\frac{8}{17}\right)$

$\sin\beta = -\frac{8}{17}$

(b) $\cos^2 30°.\tan\beta$

$= (\cos 30°)^2 . \left(\frac{-8}{-15}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 . \frac{8}{15}$

$= \frac{3}{4} . \frac{8}{15}$

$= \frac{2}{5}$

5.1.3 ref $\angle$

$\sin\beta = -\frac{8}{17}$

$\sin\theta = \frac{8}{17}$

$\theta = \sin^{-1}\left(\frac{8}{17}\right)$

$\theta = 28{,}0724...$

$\therefore \beta = 180° + \theta$

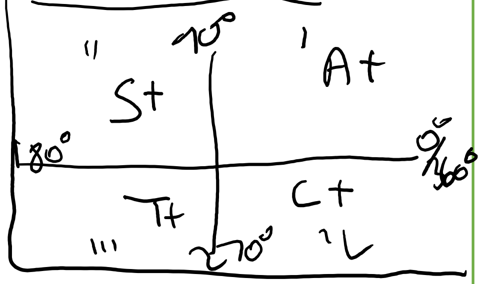
$\beta = 180° + 28{,}0724.. = 208{,}07°$

Exercise B

4.4 Given that $5\cos\beta - 3 = 0$ and $0° < \beta < 90°$.

If $\alpha + \beta = 90°$ and $0° < \alpha < 90°$, calculate the value of $\cot\alpha$.

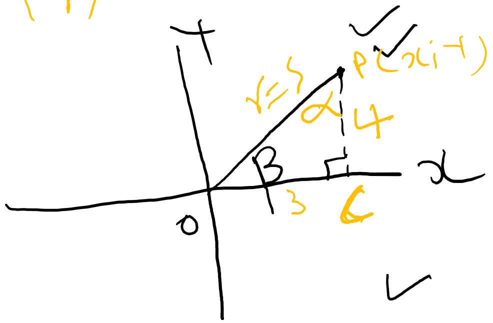
ASTC rule



Solution

$5\cos\beta - 3 = 0$

$\cos\beta = \frac{3}{5}$ $\left(\frac{x}{r}\right)$



$x^2 + y^2 = r^2$

$(3)^2 + y^2 = (5)^2$

$\sqrt{y^2} = \sqrt{16}$

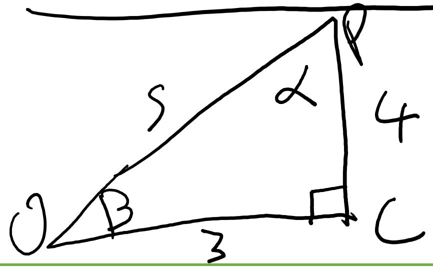
$y = \pm 4$

$\therefore y = 4$

$\alpha + \beta = 90°$

$\alpha = 90° - \beta$

In ΔOPC



$\tan\alpha = \frac{3}{4}$

$\cot\alpha = \frac{4}{3}$

Exercise C

QUESTION 4

4.1 Given $4\cot\theta + 3 = 0$ and $0° < \theta < 180°$.

4.1.1 Use a sketch to determine the value of the following. DO NOT use a calculator.

(a) $\cos\theta$

(b) $\dfrac{3\sin\theta\sec\theta}{\tan\theta}$

4.1.2 Hence, show that $\sin^2\theta - 1 = -\cos^2\theta$.

Exercise C

QUESTION 4

4.1 Given $4\cot\theta + 3 = 0$ and $0° < \theta < 180°$.

4.1.1 Use a sketch to determine the value of the following. DO NOT use a calculator.

(a) $\cos\theta$

(b) $\dfrac{3\sin\theta\sec\theta}{\tan\theta}$

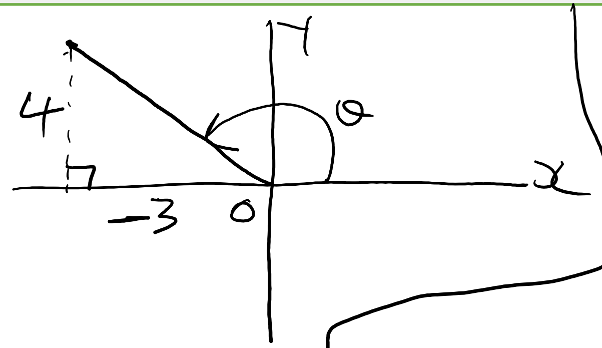
4.1.2 Hence, show that $\sin^2\theta - 1 = -\cos^2\theta$.

Solution

4.1.1 $4\cot\theta + 3 = 0$

$\cot\theta = -\dfrac{3}{4}$

$\tan\theta = -\dfrac{4}{3}$ $\left(\dfrac{y}{x}\right)$



$x^2 + y^2 = r^2$

$(-3)^2 + (4)^2 = r^2$

$25 = r^2$

$\sqrt{r^2} = \sqrt{25}$

$r = \pm 5$

$\therefore r = 5$

$\cos\theta = \dfrac{-3}{5}$

$\cos\theta = -\dfrac{3}{5}$

b) $\dfrac{3\sin\theta\sec\theta}{\tan\theta}$

$= \dfrac{\cancel{3} \times \left(\dfrac{4}{\cancel{5}}\right) \times \left(-\dfrac{\cancel{5}}{\cancel{3}}\right)}{\left(-\dfrac{4}{3}\right)}$

$= (-4) \div \left(-\dfrac{4}{3}\right)$

$= (\cancel{4}) \times \left(-\dfrac{3}{\cancel{4}}\right)$

$= 3$

4.1.2 $L.H.S = \sin^2\theta - 1$

$= \left(\dfrac{4}{5}\right)^2 - 1$

$= -\dfrac{9}{25}$

$R.H.S = -\cos^2\theta$

$= -\left(-\dfrac{3}{5}\right)^2 = -\dfrac{9}{25}$

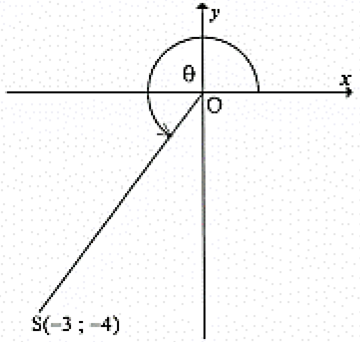
$\therefore L.H.S = R.H.S$

UBM

Trigonometry (Definitions Extended for $0° \le \theta \le 360°$) - GRADE 10
Exercises - D

Exercise D

3.2 S(−3 ; −4) is a point on the Cartesian plane such that OS makes an angle of θ with the positive x-axis.



Calculate the following WITHOUT using a calculator:

3.2.1 The length of OS

3.2.2 The value of $\sec\theta + \sin^2\theta$

Solution

3.2.1 $x^2+y^2=r^2$

$(-3)^2+(-4)^2=r^2$

$25=r^2$

$\sqrt{r^2}=\sqrt{25}$

$r=5$

$\therefore OS = 5$ units

3.2.2 $\sec\theta+\sin^2\theta$

$=\left(-\frac{5}{3}\right)+\left(-\frac{4}{5}\right)^2$

$=\frac{-5}{3}+\frac{16}{25}$

$=\frac{-5}{3}\times\frac{25}{25}+\frac{16}{25}\times\frac{3}{3}$

$=\frac{-125}{75}+\frac{48}{75}$

$=\frac{-125+48}{75}$

$=-\frac{77}{75}$

END

$$e^{i\pi}+1=0$$

Euler's Identity

SOURCES

1. *FET CAPS DOCUMENT*
2. *GRADE 10 EXAMINATION GUIDELINES*
3. *GRADE 10 DBE/NOVEMBER 2015 -2018*